Hole doping and disorder effects on the one-dimensional Kondo lattice, for ferromagnetic Kondo couplings

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Abstract

We investigate the one-dimensional Kondo lattice model (1D KLM) for ferromagnetic Kondo couplings. The so-called ferromagnetic 2-leg spin ladder and the S=1 antiferromagnet occur as new one-dimensional Kondo insulators. Both exhibit a spin gap. But, in contrast to the strong coupling limit, the Haldane state which characterizes the 2-leg spin ladder Kondo insulator cannot fight against very weak exterior perturbations. First, by using standard bosonization techniques, we prove that an antiferromagnetic ground state occurs by doping with few holes; it is characterized by a form factor of the spin-spin correlation functions which exhibits two structures respectively at $q = \pi$ and $q = 2k_F$. Second, we prove precisely by using renormalization group methods that the Anderson-localization inevitably takes place in that weak-coupling Haldane system, by the intoduction of quenched randomness; the spin-fixed point rather corresponds to a "glass" state.

Finally, a weak-coupling "analogue" of the S=1 antiferromagnet Kondo insulator is proposed; we show that the transition into the Anderson-localization state may be avoided in that unusual weak-coupling Haldane system.

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I. INTRODUCTION:

The Kondo lattice model (KLM) consists of conduction electrons magnetically coupled to a spin array through the Kondo interaction. This model is ruled by two important parameters, namely the hopping strength t of the conduction electrons, and the Kondo coupling λ_k which can be ferromagnetic or antiferromagnetic. This model has been actively studied in the context of an antiferromagnetic Kondo coupling. Empirically, Kondo lattices are described by a large and positive ratio λ_k/t, they resemble metals with very small Fermi energies of the order of several degrees. It is widely believed that conduction and localized electrons in the Kondo lattice hybridize at low-temperature to create a single narrow band. But understanding of this process remains vague and it is not clear whether the localized electrons contribute to the volume of the Fermi sea. The available experimental data apparently support the fact that at low temperatures, this class of compounds behave as semiconductors with very small gaps [1]. From another point of view, the compounds which show a small and positive ratio λ_k/t are generally described by a magnetic ground state, generated by conduction electron exchange or the so-called Ruderman-Kittel-Kasuya-Yosida (RKKY) interaction [2]. In that case, RKKY interactions induce an antiferromagnetic spin ordering transition at a temperature of order λ_k^2/t before the Kondo effect has the opportunity to quench the local moment. Finally, for more general and positive ratios λ_k/t [3] the competition between the RKKY interaction and the Kondo effect leads generally to heavy-fermion systems like $CeAl_3$ [4], which correspond to Fermi liquid systems on the brink of magnetism.

For some years, quantum effects in low-dimensional antiferromagnetic spin systems have also attracted theoretical and experimental interest. One such effect is the formation of a spin gap, which competes with antiferromagnetic long-range order instabilities. The origin of the spin gap in relation to the structural features has been extensively studied in a unified way for various systems. An example of such a system is the spin-1 antiferromagnetic Heisenberg chain which behaves quite differently from a spin-1/2 chain and which exhibits

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the so-called Haldane gap [5]. The ground state of the spin-1 chain, with an open boundary condition has a novel string-topological order [6] and resembles a Valence Bond Solid (VBS) state [7]. A typical S=1 quasi one-dimensional antiferromagnet $Ni(C_2H_8N_2)_2NO_2(ClO_4)$ (NENP) exhibits spin gap behavior [8] and provides experimental evidence to support the existence of a VBS state. In this material, the impurity effect plays an important role as a probe for investigating the microscopic properties. Another compound which shows spin-gap behavior is the 2-leg spin-ladder system, $SrCu_2O_3$ [9]. The origin of the spin gap is understood using the short-range Resonating Valence Bond (RVB) picture [10,11]. Recently, experimental studies on Z_n-doped compounds $Sr(Cu_{1-x}Z_{n_x})_2O_3$ [12] have been performed. The experimental results suggest that, for $x \geq 0.01$ there is an antiferromagnetic phase at low temperatures. The phenomena observed in the case of $Sr(Cu_{1-x}Z_{n_x})_2O_3$ suggest, that unlike the case of impurity doping in NENP, a substantial change occurs in the bulk spin state with Z_n doping of less than 1%.

In this article, we investigate the case of the 1-dimensional KLM (1D KLM) for a *ferromagnetic* Kondo coupling. In this context, we will show that the so-called ferromagnetic 2-leg spin ladder and the S=1 antiferromagnet systems occur as new interesting Kondo insulators. Then, we will investigate the effects of very weak exterior perturbations, in these two similar Kondo insulators. More precisely, by applying standard bosonization methods [13–15] and renormalization group methods, we will prove that a substantial change occurs in the bulk spin state of the 2-leg spin ladder Kondo insulator by the introduction of few holes, and also by the application of quenched disorder. In the strong coupling regime, the problem lacks a small parameter and cannot be analyzed by perturbation theory. But, we will give some indisputable qualitative and topological arguments to check that in that case, the Haldane phase is rather stable against the same weak exterior perturbations.
Finally, we introduce a weak-coupling "analogue" of the S=1 antiferromagnet Kondo insulator, and we will prove that the transition into the Anderson localization state [16] may be suppressed in that unusual weak-coupling Haldane system.

II. PURE 1D KLM, FOR FERROMAGNETIC KONDO COUPLINGS

We, first, consider the pure 1d KLM:

$$\mathcal{H} = -t \sum_{i,\sigma} c^\dagger_{i,\sigma} c_{i+1,\sigma} + U \sum_{i,\sigma} n_{i,\sigma} n_{i,-\sigma}$$
$$+ \lambda_k \sum_i \vec{S}_{f,i}.c^\dagger_i \frac{\vec{\sigma}}{2} c_i + \lambda_f \sum_i \vec{S}_{f,i}.\vec{S}_{f,i+1} \tag{1}$$

with: $(\lambda_k < 0 , \lambda_f > 0 , U > 0)$.

Here, $c^\dagger_{i,\sigma}$ $(c_{i,\sigma})$ creates (annihilates) an electron of spin σ at site i and $\vec{S}_f$ is a spin $\frac{1}{2}$ operator. The term λ_k describes the (anisotropic) ferromagnetic Kondo coupling and the term U models the so-called Hubbard interaction between electrons. We also include an explicit nearest neighbour spin coupling, λ_f. This (and longer range terms) would be generated by conduction electron exchange but it could also arise due to other direct exchanges between S_f-spins of the lattice.

We use a continuum limit of the above Hamiltonian and we linearize the dispersion of electrons. Then, we remember the following conventions used on the Minkowskian space. The relativistic fermions $c_\sigma(x)$ are separated in left-movers, $c_{L\sigma}(x)$ and right-movers, $c_{R\sigma}(x)$ on the light-cone. By using the bosonization method, the charge current $J_{c,L} = c^\dagger_{L\sigma} c_{L\sigma}(x)$ and spin current $\vec{J}_{c,L} = c^\dagger_{L\alpha} \frac{\vec{\sigma}_{\alpha\beta}}{2} c_{L\beta}$ operators can be respectively depicted by a scalar field Φ^c_c and a SU(2) matrix g [15]. For the charge sector, it is convenient to remember the identifications: $J_{c,R} + J_{c,L} = \frac{1}{\sqrt{\pi}} \partial_x \Phi^c_c$ and $J_{c,R} - J_{c,L} = -\frac{1}{\sqrt{\pi}} \Pi^c_c$, where Π^c_c is the moment conjugate to the field Φ^c_c.

Obviously, to bosonize the Kondo interaction, we need the bosonized representation for the conduction spin operator and the localized spin operators, $\vec{S}_f$; they are given [15]:

$$c^\dagger(x) \frac{\vec{\sigma}}{2} c(x) \simeq \vec{J}_{c,L}(x) + \vec{J}_{c,R}(x) \tag{2}$$
$$+ \text{constant}. \exp(i2k_F x) tr(g.\vec{\sigma}) \cos(\sqrt{2\pi}\Phi^c_c)$$
$$\vec{S}_f \simeq \vec{J}_{f,L}(x) + \vec{J}_{f,R}(x) + \text{constant}.(-1)^x tr(f.\vec{\sigma})$$

Finally, the bosonized Hamiltonian reads:

$$\mathcal{H} = H_c + H_s + H_k \tag{3}$$

with:

$$H_c = \int dx \, \frac{u_\rho}{2K_\rho} : (\partial_x \Phi_c^c)^2 : + \frac{u_\rho K_\rho}{2} : (\Pi_c^c)^2 : + g_3 \cos(\sqrt{8\pi}\Phi_c^c) \tag{4}$$

$$H_s = \frac{2\pi v}{3} \int dx \, : \vec{J}_{c,L}(x)\vec{J}_{c,L}(x) : + \frac{2\pi v_f}{3} \int dx \, : \vec{J}_{f,L}(x)\vec{J}_{f,L}(x) :$$

$$+ \int dx \, \lambda_1 : \vec{J}_{c,L}(x)\vec{J}_{f,L}(x) : + (L \to R)$$

$$H_k = \int dx \, \lambda_2[\vec{J}_{c,L}(x)\vec{J}_{f,R}(x) + \vec{J}_{c,R}(x)\vec{J}_{f,L}(x)]$$

$$+ (-1)^x \exp(i2k_F x).\lambda_3 tr(g.\vec{\sigma}) tr(f.\vec{\sigma}) \cos(\sqrt{2\pi}\Phi_c^c)$$

where: $\lambda_{i=1,2,3} = (a\lambda_k)$, $g_3 = (aU)$, $v_f = (a\lambda_f)$ and $v = (at)$; a is the lattice step.
H_c describes the well-known Tomonaga-Luttinger (TL) liquid [17]. The coupling g_3 generates the usual $4k_F$-Umklapp process; the u_ρ, K_ρ parameters of the TL liquid (or the Hubbard chain) are given by:

$$u_\rho K_\rho = v \qquad \text{and} \qquad \frac{u_\rho}{K_\rho} = v + U/\pi \tag{5}$$

In the spin Hamiltonians H_s and H_k, the two spin bosons turn out to be very similar to the ones usually discussed for two spin chains; thus, we expect some resemblance between spin excitations in the 1D KLM and spin excitations in the problem of two coupled spin chains. Finally, to keep the "Lorentz invariance" of the theory we choose the particular bare conditions $\lambda_f = t$ and $\lambda_1 = 0$; we deal with a single velocity of light in the spin sector.

A. Weak-coupling limit

Since the limit to which our method applies is ($\lambda_k << t$), we begin to investigate this situation precisely. The term λ_2 generates the (anisotropic) ferromagnetic Kondo coupling. It is marginally relevant, and by using the well-known Operator Product Expansion (OPE) of a $SU(2)_{k=1}$ algebra [18,19], we deduce that it renormalizes to large values producing an exponentially small gap with the Ising anisotropy:

$$|\Delta_z| \propto (\lambda_{2,\perp} - \lambda_{2,z}) << 1 : \tag{6}$$

and:

$$m \propto \exp(-\text{constant}/\sqrt{|\Delta_z|}) \tag{7}$$

1. 2-leg spin ladder Kondo insulator

At half-filling, the $2k_F$ oscillation becomes commensurate with the alternating localized spin operator, and the term λ_3 which generates the $q = (2k_F + \pi)$ excitations is strongly relevant; in the limit $U << \lambda_k$, it obeys the renormalization flow:

$$\frac{\partial \lambda_3}{\partial lnM} = (2 - 3/2)\lambda_3 \tag{8}$$

Since it opens a mass gap $M \propto \lambda_3^2 >> m$ for the charge and all the spin excitations, we conclude that at half-filling, it rules all the (interchain) spin excitations. The ferromagnetic Kondo coupling does not renormalize. In the limit $U << M$, the Umklapp process does not change the physics either. Hence, both can be dropped out.

Now, to briefly describe the ground state and to yield clearly the influence of the topological effects on it, we begin by using the spin-charge separation phenomenon: the charge sector is massive and we note K the non-universal value defined by $K = \lambda_3 \langle \cos(\sqrt{2\pi}\Phi_c^c) \rangle$; K is proportional to $-(a\lambda_k)^2$. Since the two spin bosons $\vec{J}_c$ and $\vec{J}_f$ turn out to be very similar to the ones usually discussed for two spin chains, this variety of Kondo insulator is assumed to be very similar to a 2-leg spin ladder system; it can be viewed as a spin system, defined by a spin operator which is larger than a s=1/2 spin but smaller than a S=1 spin. We use the representation of $\vec{J}_{f,(L,R)}$ in terms of the matrix f [15]:

$$\vec{J}_{f,L} = -\frac{i}{2\pi} tr[\partial_- f f^\dagger . \vec{\sigma}] \qquad \vec{J}_{f,R} = \frac{i}{2\pi} tr[f^\dagger \partial_+ f . \vec{\sigma}] \tag{9}$$

We have introduced the light-cone coordinates; by using the analogue representation of $\vec{J}_{c,L(R)}$ in terms of the SU(2) g-matrix, one obtains the *standard* 2-leg spin-ladder Hamiltonian [20]:

$$H_s + H_k = W(f) + W(g) + \int dx \; Ktr(g.\vec{\sigma})tr(f.\vec{\sigma}) \tag{10}$$

$W(f)$ is the so-called $SU(2)_{k=1}$ Wess-Zumino-Witten Hamiltonian [18,19]:

$$W(f) = \frac{1}{4u^2} \int dx \; tr(\partial_\mu f \partial_\mu f^\dagger) + \frac{k}{24\pi} \int_0^\infty d\xi \int dx \; \epsilon^{\mu\nu\lambda} tr(f^\dagger \partial_\mu f f^\dagger \partial_\nu f f^\dagger \partial_\lambda f) \tag{11}$$

We take $v_f = 1$ in the definition of $W(f)$. The second term in the expression of $W(f)$ has a topological origin, and the corresponding charge $k = 1$ is defined "modulo 2".

At long distance, the matrices f and g are traceless; from topological and kinetic points of view, the spin problem becomes equivalent to the problem of two coupled "identical" spin chains. In that sense, the asymptotic behavior of the resulting spin system should be ruled by an Hamiltonian of type 2W(f), which is characterized by the dynamical parameter $\tilde{u} = u/\sqrt{2}$, and the topological charge $k = 2$. To prove that explicitly, we can substitute $(g = f\alpha^\dagger)$, in the above expression. Then, by using the following identity [20,18]:

$$W(g) = W(f\alpha^\dagger) = W(f) + W(\alpha) + \frac{1}{8\pi} tr \int dx \; f^\dagger \partial_- f \alpha^\dagger \partial_- \alpha \tag{12}$$

we obtain precisely that the α field-Hamiltonian is not relevant at long distance. Indeed, although $W(\alpha)$ is governed by a $k = 0$ topological charge, its kinetic term shows a less relevant dynamical parameter than 2W(f) (because $u^{-1} < \tilde{u}^{-1}$). Finally, a coupling like $(\partial_- \alpha.f)$ is not relevant at long distance either. Then, since the topological charge is defined "modulo 2", the Hamiltonian $2W(f)$ behaves as a system without any topological charge and this indisputable fact rules all the low-energy physics.

First, the dynamical coupling u is now asymptotically free and by using the well-known "background method" [21], we obtain:

$$\beta(u) = \frac{\partial u^2}{\partial lnM} = -\frac{1}{4\pi} u^4 \tag{13}$$

By the use of the relevant cut-off $M \propto \lambda_3^2$ for the spin excitations, it comes that the ground state is *confined* inside the small characteristic length:

$$\xi_H \propto \frac{u^2}{M} \exp(\frac{+4\pi}{u^2}) \tag{14}$$

Second, site Parity $x \to -x$ is not broken and as a consequence, the ground state is non-degenerate. We remember that site Parity acts on f simply as:

$$P_S : f \to -f^\dagger \tag{15}$$

Third, spin-1/2 excitations can not occur. Definitively, we obtain the same short-range RVB state which does not possess any free spinon, than in the purely two-leg spin ladder problem. Finally, lowest-energy spin excitations (well-modelled by the instantonic term K) are composed of singlet-triplet excitations; they occur by the "spin-flip" of a spinon, and generate magnetism characterized by spin-spin correlation functions decreasing exponentially. At higher energy, singlet-singlet excitations may occur due to the spin flip of two spinons; they bring corrections to spin-spin correlation functions.

2. Hole-doping effect

Even for a very small hole-doping, the $2k_F$ oscillation is not commensurate with the alternating localized spin operator, and the term λ_3 can be dropped out. The system behaves as a metallic state, and it is not well-described by any RVB state since the spin gap is destroyed by the commensurate-incommensurate transition. Inevitably, a substantial change occurs in the bulk spin state of the 2-leg spin ladder Kondo insulator, by doping with few holes. Now, it is necessary to properly re-define the ground state and the low-energy spin excitations in that situation.

The nearest neighbour λ_f-coupling favors $q = \pi$ excitations in the spin chain. The RKKY interaction, characterized by the energy scale $T_A \simeq \lambda_k^2/t$, has also to be taken into account; indeed, the energy scale T_A is more relevant than the mass gap m. It implies that the term λ_2 is not submitted to renormalization effects, and the RKKY interaction may control the short-range distance behavior either, making a characteristic structure around $q = 2k_F$ in correlation functions. In the present case, the form factor of the S_f-spin correlation function

may exhibit two structures in momentum space at $q = 2k_F$ and $q = \pi$. Then, by using general results on the TL liquid and on the spin-1/2 antiferromagnetic chain, the precise power-law for the spin-spin correlation functions are given:

$$\langle \vec{J}_f(0,0).\vec{J}_f(x,0)\rangle \propto \frac{C_1 \cos(2k_F x)}{x^{1+K_\rho}} + \frac{C_2 \cos(\pi x)}{x} \tag{16}$$

We have omitted less relevant contributions. The term C_1 comes from the magnetic polarization by the 1D electron gas; it corresponds to the usual spin-spin correlation functions in the 1D electron gas. The term C_2 is generated by the direct exchange λ_f. In the model, $\lambda_f \simeq t$ is assumed to be much larger than λ_k^2/t; hence, we deduce that $C_2 >> C_1$. The $q = \pi$ structure is expected to be more prominent than the $q = 2k_F$ one. For a small hole-doping, the incommensurate RKKY interaction enhances the antiferromagnetism generated by the Heisenberg exchange but, at quarter filling it would favor a ferromagnetic exchange between localized spins which tend to compete with it. In the limit $\lambda_f >> \lambda_k^2/t$, the antiferromagnetism wins and the $q = 2k_F$ spin excitations are assumed to vanish.

It is interesting to remark that, as in the impurity-doped Haldane system $Sr(Cu_{1-x}Zn_x)_2O_3$, the disappearance of the Haldane phase enhances much the antiferromagnetic spin-spin correlation functions in that case.

Finally, we also insist on the fact that the 1D electron gas is weakly affected by the RKKY process. More precisely, if we keep only the terms which contain the spinon field $\vec{J}_c$ (in the Hamiltonian), we obtain an Hamiltonian of the form:

$$H_s^*(c) = \frac{2\pi v^*}{3} \int \, dx \; : \vec{J}'_L(x)\vec{J}'_L(x): \, +L \leftrightarrow R \tag{17}$$

Now, the precise spinon field writes:

$$\vec{J}'_L \simeq \cos\theta \vec{J}_{c,L} + \sin\theta \vec{J}_{f,R} \tag{18}$$

and:

$$\sin\theta/\cos\theta = 3\lambda_2/2\pi v \tag{19}$$

In the limit $|\lambda_k| << t$, the angle θ is very small. It proves that the spin array does not manage to quench the 1D electron gas, in the weak-coupling limit. Accordingly, the single notable effect producing in the 1D electron gas is that the velocity of the free spinons $\vec{J}_{L(R)}$ is increased to $v^* = v/\cos^2\theta = v.(1 + \frac{9\lambda_2^2}{4\pi^2 v^2})$ due to the RKKY process; it is increased due to the magnetic diffusion by the spin array. Finally, since no charge fluctuation is authorized in the spin array, the charge sector of the 1D electron gas is not affected; it is always ruled by the boson field Φ_c^c.

B. Strong coupling limit

1. S=1 antiferromagnet Kondo insulator

When $|\lambda_k| \geq t$, the OPE used to obtain eqs.(7) and (8) breaks down in this limit, and therefore the problem cannot be analyzed in perturbation theory. However, since the two spin bosons $\vec{J}_c$ and $\vec{J}_f$ turn out to be very similar to the ones usually discussed for two spin chains, the ground state is expected to be very similar to a pure S=1 antiferromagnet system. Instead, a variety of approximate methods have been suggested to study the S=1 antiferromagnetic spin chain [7,22]. But, it is not our purpose to review them here. We give some topological arguments to confirm that the so-called Valence Bond Solid (VBS) state corresponds to the ground state of the 1D KLM in that strong coupling regime. We remember that, in the VBS state each spin-1 is composed of two symmetrized spin-1/2 objects, and the spin-1/2 objects form singlets with the spin-1/2 objects on neighboring sites, resembling rather the ground state of a dimerized spin-1/2 chain.

Here, the SU(2) spin symmetry of the electronic spin operator is explicitly broken onto $U(1) \times \mathcal{Z}_2$, and the $\mathcal{Z}_2$ symmetry which corresponds to $\sigma^z \rightarrow -\sigma^z$ is spontaneously broken due to the influence of the term λ_2. We expect:

$$\langle g_\beta^\alpha \rangle \propto (\sigma^z)_\beta^\alpha \longrightarrow \langle S_i^z \rangle \propto \pm 1. \tag{20}$$

First, we may conclude that the above short-range RVB description is certainly not available in that case; "interchain" singlet valence bonds could not occur, and it proves that the term λ_3 becomes meaningless in the strong coupling limit. Second, as in the strong antiferromagnetic Kondo coupling limit, the strong ferromagnetic Kondo coupling tends to quench the 1D electron gas. Unlike the weak-coupling regime, we expect a large charge gap which varies like $|\lambda_k^z|/t$; obviously, the Umklapp process does not change anything about this conclusion. Finally, in contrast to the previous case, the low-energy physics cannot be described by spin-1 objects. Indeed, in that case, the ground state is still submitted to the fact that we couple two level-1 $SU(2)$ WZW theories in the spin sector [22]. It means that the asymptotic behavior of the spin-1 system is also ruled by an Hamiltonian of type $2W(f)$; the relevant topological charge is $k = 2$ and the dynamical coupling is $\tilde{u} = u/\sqrt{2v_f}$. We deduce that, the ground state of the S=1 antiferromagnet Kondo insulator, is also non-degenerate and especially it is also described by a sea of singlet valence bonds. Of course, since "interchain" valence bonds could not occur in the strong coupling limit, the latter are now formed separately in the TL liquid and in the spin array. It produces a non-zero Haldane gap in the excitation spectrum but in that case, it is of the same order of magnitude $\lambda_f = t$. We also remember that in the weak-coupling limit, we have fixed the large velocity v_f of the free spinons to 1; in the strong coupling regime, the 1D electron gas is rather trapped by the spin array, and it leads to impose $v_f \ll 1$. With the used formalism, it consists to renormalize, $\tilde{u} \to Nu$ with $N \to +\infty$; the dynamical parameter $\tilde{u}$ is free at high energy, and eq.(13) is not correct: the ground state has no finite characteristic length, and $\xi_H \to +\infty$. Definitively, we conclude that the ground state resembles a VBS state.

2. Hole doping effect

Away from half-filling, localization remains in the transport properties due to the large charge gap of order $|\lambda_k^z|/t$ imposed by the term λ_2, and the weak-coupling treatment obviously breaks down (it would correspond to $\theta > \frac{\pi}{4}$). Here, since the massless charge exci-

tations are considerably reduced by the magnetic interaction with the spin array, we argue that the ground state of a hole-doped S=1 antiferromagnet Kondo insulator is very similar to the ground state of an impurity-doped S=1 antiferromagnet. Thus, we can use some results concerning the effects of non-magnetic impurities driven on a S=1 antiferromagnet, to investigate hole-doping effects on the S=1 antiferromagnet Kondo insulator.

In the case of a very small hole doping, by using ref. [23], we conclude that the Haldane phase and its topological structure should be quite stable against weak-bond randomness. Low-energy excited states below the spin gap may appear upon hole-doping due to the presence of few free spinons, but the Haldane gap is not "filled" in the context of weak-bond randomness.

For larger doping regions, the Haldane state cannot survive anymore; it can not fight against an important number of free spinons. The topological features become obviously badly defined because the spin system shows now a large number of residual s=1/2-spins. It is very difficult to investigate theoretically such a spin system; we may treat only the other limit of a very small electronic density of state. In that context, the resulting spin system consists of a spin-1/2 antiferromagnet where some rare S=1 impurities are included. From a topological point of view, the presence of a single S=1 magnetic impurity disturbs locally the spin-1/2 chain. Accordingly, we expect to flow to the open spin-1/2 chain fixed point [24]. More precisely, to stabilize the system, the S=1 magnetic impurity tends to couple antiferromagnetically to the next pair of spins in the chain (via the Kondo effect) and consequently, the antiferromagnetic couplings to the screening spins give a purely S=0 singlet which cuts the chain.

Finally, due to the narrow link between charge and spin excitations in the 1D KLM, we have proved that in contrast to the weak-coupling limit, the Haldane phase which describes the strong coupling regime is still stable against a very small hole-doping. It is due to the important fact, that massless charge excitations are considerably suppressed by the strong Kondo coupling; charge excitations are not prominent, that allows to still stabilize the VBS state in the presence of few holes. To summarize, a schematic phase-diagram of the (pure)

1D KLM, in the case of a ferromagnetic Kondo coupling is proposed in Figure. 1.

III. WEAK COUPLING LIMIT: INFLUENCE OF QUENCHED DISORDER

Now, since the 1D KLM couples charge and spin degrees of freedom, it is relevant to investigate randomness effects on these two similar Kondo insulators. First, the S=1 anti-ferromagnet Kondo insulator should not be really affected by the presence of defects since it opens a large charge gap of order $|\lambda_k^z|/t$. The 1D electron gas is quenched by the spin array and backward scatterings due to impurities appear irrelevant. Second, we do not know randomness effects in the two-leg spin ladder Kondo insulator. To investigate precisely this point, it is necessary to switch over to Abelian bosonization notation.

A. Abelian representation

The SU(2) field g can be written in terms of the free spin bosons: $\Phi_c = \Phi_{c,L} + \Phi_{c,R}$ and its dual $\tilde{\Phi}_c = \Phi_{c,L} - \Phi_{c,R}$.

$$
g = \begin{pmatrix} \exp(i\sqrt{2\pi}\Phi_c) & \exp(i\sqrt{2\pi}\tilde{\Phi}_c) \\ -\exp(-i\sqrt{2\pi}\tilde{\Phi}_c) & \exp(-i\sqrt{2\pi}\Phi_c) \end{pmatrix} \tag{21}
$$

For the SU(2) field f, we introduce the free bosons: $\Phi_f = \Phi_{f,L} + \Phi_{f,R}$ and its dual $\tilde{\Phi}_f = \Phi_{f,L} - \Phi_{f,R}$. It is useful to work with the two linear combinations: $\Phi_\pm = (\Phi_c \pm \Phi_f)/\sqrt{2}$ and their canonical conjugate momenta $\Pi_\pm = \partial_x \tilde{\Phi}_\pm$. Then, we obtain:

$$
\begin{aligned}
H_s + H_k = \sum_{\nu=+,-} \int dx \, \frac{u_\nu}{2K_\nu} : (\partial_x \Phi_\nu)^2 : + \frac{u_\nu K_\nu}{2} : (\Pi_\nu)^2 : \\
+ \int dx \, [\lambda_4 \cos(\sqrt{4\pi}\Phi_-) \\
+ 2\lambda_5 \cos(\sqrt{4\pi}\tilde{\Phi}_-) - \lambda_6 \cos(\sqrt{4\pi}\Phi_+)] \cos(\sqrt{2\pi}\Phi_c^c)
\end{aligned} \tag{22}
$$

where $\lambda_{i=4,5,6} \propto \lambda_3$. The Kondo term λ_2 has not been introduced since it is not relevant, here. In the charge sector, the Umklapp process is useless in the limit $U << \lambda_k$; it does not play any role in the following discussions.

B. Renormalization flow

Now, we may investigate precisely the effects of quenched disorder in this interesting Kondo insulator. We apply renormalization group methods, first used by Giamarchi and Schulz [25], in the context of randomness in a TL liquid. We introduce the complex random impurity potential,

$$H_{imp} = \sum_\sigma \int dx \ \xi(x) c_{\sigma L}^\dagger c_{\sigma R} \qquad + hc \qquad (23)$$

with the Gaussian distribution:

$$P_\xi = \exp(-D_\xi^{-1} \int dx \ \xi^\dagger(x)\xi(x)) \qquad (24)$$

We can omit forward scatterings, because the q=0 random potential just renormalizes the chemical potential and it does not affect the fixed point properties or equivalently the staggered part of the electronic spin operator.

In order to deal with the quenched disorder, we use the well-known *replica* trick [26]. Due to the definition of $\xi(x)$, we are limited to a weak randomness treatment and we include only the first contribution (order) in D_ξ. Then, there is no coupling between different replica indices, which will be omitted below. In its bosonized form, the Hamiltonian H_{imp} reads:

$$H_{imp} \simeq \frac{1}{2\pi a} \int dx \ \xi(x) \exp^{i(\sqrt{2\pi}\Phi_c^c - 2k_F x)} \{\cos\sqrt{2\pi}\Phi_+(x) + \cos\sqrt{2\pi}\Phi_-(x)\} + hc \qquad (25)$$

By applying a standard renormalization group analysis up to the lowest order in λ_k and D_ξ, we obtain the complete flow:

$$\frac{dD_\xi}{dl} = (3 - \frac{K_+}{2} - \frac{K_-}{2} - K_\rho)D_\xi \qquad (26)$$

$$\frac{d\lambda_4}{dl} = (2 - K_- - \frac{K_\rho}{2})\lambda_4 \qquad (27)$$

$$\frac{d\lambda_5}{dl} = (2 - \frac{1}{K_-} - \frac{K_\rho}{2})\lambda_5 \qquad (28)$$

$$\frac{d\lambda_6}{dl} = (2 - K_+ - \frac{K_\rho}{2})\lambda_6 \qquad (29)$$

$$\frac{dK_+}{dl} = -\frac{1}{2}(D_+ + \lambda_6^2)K_+^2 \qquad (30)$$

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$$\frac{dK_-}{dl} = -\frac{1}{2}(D_- + \lambda_4^2)K_-^2 + 2\lambda_5^2 K_-^2 \tag{31}$$

$$\frac{dK_\rho}{dl} = -\frac{1}{2}u_\rho[(\frac{D_-}{u_-} + \frac{D_+}{u_+}) + (\frac{(\lambda_4^2 + 4\lambda_5^2)}{u_-} + \frac{\lambda_6^2}{u_+})]K_\rho^2 \tag{32}$$

with the notations: $dl = d\mathrm{Ln}L$, $D_\nu = 2\frac{D_\xi a}{\pi u_\nu^2}[\frac{u_\nu}{u_\rho}]^{K_\rho}$ and $\tilde{\lambda}_\nu = \frac{\lambda_\nu}{2\pi u_\nu}$, and where we did not display equations irrelevant to the following discussions. We particularly notice that in this problem, there is no term like $D_\xi \lambda_{(4,6)}$ generated by perturbation.

C. Disorder effects on the 2-leg spin ladder Kondo insulator

In the 2-leg spin ladder situation, the spin excitations are quite isotropic and the initial conditions on the pure system are:

$$K_+(0) \simeq 1, \qquad K_-(0) \simeq 1, \qquad K_\rho(0) \leq 1 \tag{33}$$

Then, in the pure system, we initially have: $(2 - K_- - \frac{K_\rho}{2}) > 0$, $(2 - \frac{1}{K_-} - \frac{K_\rho}{2}) > 0$ and $(2 - K_+ - \frac{K_\rho}{2}) > 0$. We start with a small initial parameter $\lambda_5(0)$ as $D_\xi(0)$, and we investigate the fixed point properties. By using eqs.(26) and (28), we may immediately remark that D_ξ scales to the strong coupling regime before the couplings λ_ν; indeed, in that case $(3 - \frac{K_+}{2} - \frac{K_-}{2} - K_\rho) > (2 - \frac{1}{K_-} - \frac{K_\rho}{2})$. In the strong-coupling regime, our weak-coupling treatment for randomness breaks down. But, although in this case we do not know precisely the fixed point, it is expected that sufficiently strong disorder may destroy the TL liquid state and bring about the transition into the Anderson localization state. Now, we give physical arguments about this conclusion.

In the pure system, the presence of the Haldane gap reduces the number of massless excitations. In fact, the ferromagnetic Kondo coupling induces (magnetic) backward scatterings due to the spin array of the form:

$$|a\lambda_k|^3 (-1)^x \exp^{i2k_F x} \cos(\sqrt{2\pi}\Phi_c^c) \tag{34}$$

It develops a relevant "$2k_F + \pi$" charge-density wave (CDW). The order parameter is given by $O_{CDW}(x) = (-1)^x \langle c_{L\sigma}^\dagger(x)c_{r\sigma}(x)\rangle$. The correlation functions of this order parameter

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shows algebraic decay; $\langle O_{CDW}(x)O_{CDW}(0)\rangle = x^{-K_\rho/2}$. To discuss the eventual pinning of this CDW (or of the 1D electron gas) by randomness, we have to compare the charge gap imposed by the spin array and the strength of disorder; here, the ferromagnetic Kondo coupling opens a charge gap $M \propto |\lambda_5|^2$ which is not sufficiently large to fight against randomness: $D_\xi(0) >> M(0)$. In this context, the 1D electron gas is more attractive by the defects than by the localized spins; in that situation, the disorder becomes strongly relevant and, the Haldane state cannot prevent impurity potential from pinning the CDW. Definitively, the charge fixed point is the same than for repulsive interaction in the 1D Hubbard model [25]; the Anderson localization by disorder inevitably takes place in this weak-coupling Kondo system. Since "$2k_F + \pi$" charge and spin excitations are strongly coupled in the 1D KLM model, we conclude that the Haldane phase is also destroyed by the pinning of the CDW by the defects. Finally, the spin array and the Hubbard chain are decoupled at the fixed point, and we have to re-discuss the magnetic exchange between the localized spins in the presence of randomness. At first sight, $\lambda_f \propto t$ is sufficiently strong that a site randomness should not affect it so much; hence, the massless spin mode can survive. However, we have to additionally include randomness for the exchange λ_f interaction and finally, the system may be of the "glass" state.

To achieve disorder effects on the weak-coupling regime, we address the following question: is there a possibility to suppress the Anderson-localization transition in the weak-coupling limit? To discuss that, we propose a weak-coupling "analogue" of the S=1 antiferromagnet Kondo insulator, and we re-discuss the localization by disorder in that context.

D. Suppression of the Anderson-localization in the weak-coupling regime?

Now, we introduce a weak-coupling Haldane system, where we suppress any singlet mode in the excitation spectrum. To model the latter, we reconsider the Hamiltonian of eq.(22) but now suppress the basal spin excitations of type $tr(g.\sigma^+)tr(f.\sigma^-) \propto \cos(\sqrt{4\pi}\tilde{\Phi}_-)$. In the present case, the massive S=1 spin excitations are reduced to the doublet $S^z = \pm 1$. At

long distances, it might be expected that this weak-coupling system behaves exactly as a true S=1 antiferromagnet. To re-discuss the Anderson localization problem in that case, it is relevant to properly re-investigate the charge sector. Here, the operator $tr(g.\sigma^+)tr(f.\sigma^-)$ can be replaced by its expectation value (which is independent of λ_k). Therefore, the term λ_5 behaves as an operator of scaling dimension $1/2$ and finally, it generates (magnetic) backward scatterings of the form:

$$\lambda_5(-1)^x \exp^{i2k_Fx} \cos(\sqrt{2\pi}\Phi_c^c) \tag{35}$$

Now, the ferromagnetic Kondo coupling opens a strongly relevant charge gap: $M' \propto |\lambda_5|^{2/3}$ which is considerably larger than the Haldane gap. This weak-coupling Haldane system can be considered as a good weak-coupling "analogue" of the S=1 antiferromagnet Kondo insulator. Here, if we start with quite identical values of $\lambda_5(0)$ and $D_\xi(0)$, the Haldane state opens a charge gap $M'(0) >> D_\xi(0)$; we argue that the transition into the Anderson-localization state should be prevented in that case. To check that precisely, we give the initial conditions on this pure weak-coupling system:

$$K_+(0) \simeq 1, \qquad K_-(0) >> 1, \qquad K_\rho(0) < 1 \tag{36}$$

Then, if we start with quite identical values of $\lambda_5(0)$ and $D_\xi(0)$, it is immediate to conclude that the term λ_5 scales to strong coupling values before the disorder; here, we have $(3 - \frac{K_+}{2} - \frac{K_-}{2} - K_\rho) << (2 - \frac{1}{K_-} - \frac{K_\rho}{2})$. It shows clearly that the (magnetic) backward scatterings due to the spin array become sufficiently important to suppress the influence of the backward scatterings due to impurities. By using eq.(26) we can remark, that the disorder field D_ξ is now characterized by a negative scaling dimension $\Delta = (3 - \frac{K_+}{2} - \frac{K_-}{2} - K_\rho)$; we can check that $D_\xi \to 0$. The system is then mainly ruled by the very large parameter $K_- \to +\infty$, that shifts λ_4 to zero. Conversely, the term λ_6 remains relevant, and $K_+ \to 0$ is then renormalized to zero. This term generates the singlet-doublet spin excitations of type $S^z = \pm 1$. It confirms that the Haldane phase is not destroyed by randomness in that case; the spin gap is given by $M \propto \lambda_6^2$. Finally, K_ρ tends to zero due to the influence of

the term λ_5 and remarkably, as in the strong coupling limit, the Anderson localization is suppressed due to the magnetic localization by the spin array: it consists of a magnetic type of de-pinning effect of the CDW.

In contrast to the 2-leg spin ladder Kondo insulator, we have precisely shown that a weak-coupling "analogue" of the S=1 antiferromagnet in this model, manages to suppress the transition into the Anderson-localization state. Indeed, reduce the S=1 spin excitations occurring in the 2-leg spin ladder Kondo insulator to the doublet $S^z = \pm 1$, increases considerably the charge gap of the 1D electron gas, imposed by the spin array. The 1D electron gas becomes more attractive by the spin array than by impurities, that allows to suppress the influence of backward scatterings due to impurities, in the weak-coupling regime. Although our discussions here for the Anderson localization rely on weak-coupling renormalization-group methods, we think that the qualitative features should not be changed even if we take into account higher order correction in $(a\lambda_k)$ and D_ξ. Finally, results concerning disorder effects in the weak-coupling limit, are summarized in the schematic phase-diagram proposed in Figure. 2.

IV. CONCLUSION

Summarizing, the 2-leg spin ladder and the S=1 antiferromagnet occur as new interesting Kondo insulators; both are described by a Valence Bond state and massive singlet-triplet excitations. In this paper, it has been proved for the first time that they respond differently to a very small hole doping or quenched disorder, in the 1D KLM.

In the strong coupling limit, a narrow equivalence occurs between the breaking of the discrete spin symmetry $(\sigma^z \rightarrow -\sigma^z)$ and the pinning of the 1D electron gas by the spin array. It allows the Haldane state to be quite stable against a very small hole-doping, although low-energy excited sates below the spin gap may occur due to the presence of the rare free spinons, and either against the presence of defects in the system.

Conversely, we have shown precisely that a substantial change occurs in the bulk spin state

of the 2-leg spin ladder Kondo insulator in the presence of few holes, and also in the presence of defects. First, in the presence of few holes, the charge sector becomes inevitably massless in the weak-coupling limit. The TL liquid is now governed by free spinons, and the Haldane state is immediately destroyed. That produces an antiferromagnetic ground state, characterized by a form factor of the spin-spin correlation functions which exhibits two structures respectively at $q = \pi$ and $q = 2k_F$. Second, in the presence of quenched randomness, the transition into the Anderson-localization state inevitably takes place. Indeed, in the pure system, the ferromagnetic Kondo coupling opens a small charge gap of order the Haldane gap; that does not allow to suppress the influence of backward scatterings due to the defects. Finally, a weak-coupling "analogue" of the S=1 antiferromagnet Kondo insulator has been proposed. In that unusual weak-coupling system, due to the reduction of the massive S=1 spin excitations to the doublet $S^z = \pm 1$, the charge gap imposed by the spin array becomes much larger than the spin gap. Accordingly, it has been proved precisely that the transition into the Anderson-localization state is suppressed in that system, in presence of randomness; the 1D electron gas becomes more attractive by the spin array than by the defects. From this point of view, the Haldane state prevents impurity potential from pinning the CDW. Remarkably, in the weak-coupling regime, suppress planar spin excitations tends also to suppress massless charge excitations; we conclude that as in the strong coupling limit, the Anderson-localization by disorder becomes replaced by a magnetic localization by the spin array.

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FIGURE CAPTIONS

Fig. 1: Schematic phase diagram of the 1D KLM, for ferromagnetic Kondo couplings as a function of the number n of electrons per site.

Fig. 2: Schematic phase diagram which reports the influence of a quenched disorder on the two-leg spin ladder Kondo insulator (ruled by $K_-(0) \simeq 1$) and on a weak-coupling "analogue" of the S=1 antiferromagnet Kondo insulator (ruled by $K_-(0) >> 1$). When we start with a too strong disorder, the spin chain is immediately broken; the defects overlap, and the 1D electron gas is localized by the latter.